

## Research on FAST Ideal Paraboloid

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### Abstract

According to the isotropy of the reference sphere and the effective receiving domain of the feed cabin, the three-dimensional space problem is simplified to two-dimensional space for analysis, that is, to find the "ideal parabola". The measurement standard of "ideal parabola" is mainly the minimum total radial expansion and contraction of the actuator. At the same time, when the relative position between the apex of the parabola and the reference sphere changes, the focal distance of the parabola also changes. And the radial expansion and contraction of the actuator shall not be greater than  $\pm 0.6$ . After comprehensive consideration, the expression of two-dimensional parabola is obtained. Further rotating the parabola around the z-axis can restore it to a three-dimensional ideal paraboloid.

### Keywords

Dimension transformation method; Actuator radial expansion; Rotation matrix.

### 1. Introduction

The five hundreded meter aperture spherical radio telescope (fast) is currently the radio telescope with the largest aperture and the highest sensitivity, including the active reflector system composed of main cable net, reflection panel, lower cable, actuator and relevant supporting structures, signal receiving system (feed source cabin) and relevant control Measurement system.

Active reflector can be divided into reference state and working state. In the reference state, the reflecting surface is called the reference sphere, with a radius of about 300m and a diameter of 500m; In the working state, the reflector is called the working paraboloid, which is an approximate rotating paraboloid whose reference sphere is adjusted to a 300m aperture. The feed cabin can receive signals in a circular area with its own center and a diameter of one meter. Through the cooperation of the lower cable and the actuator, the reflecting surface is adjusted to a working paraboloid, so as to reflect and converge the parallel electromagnetic wave from the target celestial body to the area that can be received by the feed cabin.

The ideal paraboloid is a working reflector with the best effect that the celestial electromagnetic wave can be received by the feed cabin after being reflected by adjusting the radial expansion amount of the actuator under the adjustment constraint of the reflection panel.

We need to establish a spatial coordinate system with the center C of the reference sphere as the coordinate origin. Azimuth of celestial body s relative to C  $\alpha = 0^\circ$ , elevation  $\beta = 90^\circ$ , the expression of the ideal paraboloid is determined considering the adjustment factors of the reflector surface.

## 2. Analysis and Assumptions

### 2.1. Problem Analysis

To determine the ideal paraboloid, the radial displacement of each main cable node actuator must be calculated first. When the datum sphere is deformed into a working paraboloid, the displacement change of the main cable node in each direction is determined by the deformation mode of the working paraboloid, and the determination of the deformation mode is mainly based on:

- (1) The radial displacement of the main cable node is the smallest;
- (2) When the reference sphere is deformed into a working paraboloid, the difference of arc length is the smallest;
- (3) When the reference sphere is deformed into a working paraboloid, the difference of ring length is the smallest;
- (4) The diameter of the working paraboloid is 300m by default, and the connection between its edge and the datum sphere is regarded as a smooth transition.

We should determine some feasible deformation methods according to the above four points, so as to obtain the radial displacement of each actuator, and select the group with the smallest error and the smallest radial expansion of the total actuator after comparison.

Secondly, the reference sphere and the effective receiving plane of the feed cabin are highly symmetrical and isotropic in space. Therefore, the problem can be simplified by transforming the problem in three-dimensional space into two-dimensional plane, that is, parabola instead of paraboloid and arc instead of reference sphere. First get the "ideal parabola", and then restore the parabola to the three-dimensional space with MATLAB software to get the ideal paraboloid.

### 2.2. Model Assumptions

1. The thickness of the reflector is ignored.
2. The propagation of electromagnetic wave signal is not affected by other factors.
3. The shape of the feed cabin is an ideal circle.
4. All main cable nodes are located on the datum sphere.
5. After the adjustment of the main cable node, the distance between adjacent nodes will change slightly, and the change range will not exceed 0.07%
6. By default, the feed pod is considered a point.
7. The gap between reflective panels is ignored when calculating the reflective area
8. The actuator extends along the radial direction of the reference sphere towards the ball center, which is a positive direction.
9. Under the reference state, the radial expansion amount of the top of the actuator is 0, and its radial expansion range is  $\pm 0.6\text{m}$ .
10. The coordinates of the main cable node are used as the vertex coordinates of the corresponding reflection panel.
11. In the reference state, all main cable nodes are located on the reference sphere.

$$c_2 = a_2 + b_2. \quad (1)$$

## 3. Modelling and It's Solution

Import the coordinates of each main cable node in Annex I into Matlab software and carry out the following steps:

Step 1, According to the isotropy of the reference sphere and the effective receiving domain of the feed cabin, the three-dimensional space problem is transformed into a two-dimensional space problem. After observing the data in Annex 1, it can be seen that there are many main cable nodes on the plane, so the main cable nodes on the plane in the reference state are selected. If the original coordinates are  $(X_i, Y_i, Z_i)$ , then the point on the intercepted parabola is  $(Y_i, Z_i)$ . Draw the image using the tracing method.

Since the diameter of the working paraboloid is 300m, it is necessary to screen out the main cable nodes with the coordinate median value greater than 150. There are 24 filtered points in total. The position information of the main cable node is transformed into the polar coordinate system to calculate the radial expansion of the actuator. The position of the main cable node is recorded as  $(\theta_i, \rho_i)$ ,  $i=1,2,3,\dots,24$ .

Step 2, Since the coordinates of the main cable node A0 are  $(0,0, - 300.4)$  (see Annex I for details), but the diameter of the paraboloid is 300m by default, in order to make the model prediction results more accurate, we set a physical quantity to represent the position of the paraboloid vertex relative to the reference sphere.

Combining all the above considerations, we propose three ideal paraboloid strategies with different values (as shown in Table 1). When the H value changes, the distance between the paraboloid and the feed cabin also changes, so the focal distance of the parabola should also change.

**Table 1.** Ideal paraboloid strategy

|              | ball              | parabola                                         | Focal distance P | h (m)   |
|--------------|-------------------|--------------------------------------------------|------------------|---------|
| Strategy I   | $x^2 + y^2 = R^2$ | $y = \frac{x^2}{2 * 279.6} - 300$                | 279.6            | 0       |
| Strategy II  | $x^2 + y^2 = R^2$ | $y = \frac{x^2}{2 * 279.68734527} - 300.0436726$ | 279.6            | 0.4     |
| Strategy III | $x^2 + y^2 = R^2$ | $y = \frac{x^2}{2 * 280.54244} - 300.47122$      | 280.5424         | 0.47122 |

The selection process of the three strategies is as follows:

In strategy 1  $h= 0$ , that is, the vertex of the parabola is located on the datum arc.

For strategy 2, it is analyzed by geometry.

The circular arc and parabolic equation of the reference sphere on the surface can be expressed as:YOZ

$$\begin{cases} x^2 + y^2 = R^2 \\ x^2 + 2py + 2p(R + h) = 0 \end{cases} \quad (3)$$

The diameter of the parabola is 300m, that is,  $r = 300$ . The position of the feed cabin is the focus P of the parabola, and the coordinates are  $(0, - 160.2)$ . And the intersection of parabola and arc shall be  $(0, -160.2)$ ,  $(-150, -150\sqrt{3})$ ,  $(150, -150\sqrt{3})$ , so as to meet the restriction that the diameter of paraboloid is 300m.

The simultaneous solution is  $p = 279.67$ ,  $H = 300.035$ .

For strategy 3, according to the references, the performance of paraboloid is better when particle swarm optimization algorithm  $h = 0.47122$ . At this time,  $P = 280.54244$ .

Step 3, The coordinates of each point are successively substituted into the parabolic equation in the three strategies to obtain the theoretical coordinates of each main cable node, which are

transformed into polar coordinates and recorded as  $(\theta_j, \rho_j)$ ,  $j=1,2,3,\dots,24$ . When  $\theta_i$  and  $\theta_j$  the difference is no more than 0.001rad (about 0.057 °), calculate  $\rho_i, \rho_j$ . The difference is regarded as the radial expansion and contraction of the actuator and recorded as  $D_{ij}$ ,  $i=1,2,3$ , respectively represent strategy 1, strategy 2 and strategy 3,  $j = 1,2,3,\dots, 24$ , respectively represent the main cable node on the plane YOZ.

Step 4, Draw the radial displacement curve of each main cable node actuator under the three strategies, as shown in the right figure.

Step 5, Judge whether the expansion and contraction of each main cable node exceeds 0.6m in the three strategies. In strategy 1 and strategy 2, the maximum expansion and contraction of the actuator exceeds 0.6m, so strategy 1 and strategy 2 are not considered, and strategy 3 is selected as the ideal parabola.

Step 6, According to the formula

$$S_i = \sum_{j=1}^{24} D_{ij}, \quad i = 1,2,3 \quad (4)$$

Calculate the sum of expansion and contraction of each actuator  $S_i, i=1,2,3$ . The results are as follows (keep three decimal places):  $S_1=9.442, S_2=5.638, S_3=5.616$

To sum up, we finally choose strategy 3 as the expression of "ideal parabola"

$$y = \frac{x^2}{2 \times 280.54244} - 300.47122 \quad (5)$$

Step 7, In order to restore the parabola obtained in strategy 3 to a rotating paraboloid in three-dimensional space, we set up two methods: mathematical reduction and drawing reduction. By comparison, the final effect of drawing reduction method is better. The paraboloid is the ideal paraboloid required by problem 1, as shown in the right figure. The expression is:

$$z = \frac{x^2 + y^2}{559.2} - 300.47122 \quad (6)$$

## 4. Conclusion

The model is established with a large amount of data information such as the coordinate information of the main cable node of the actuator, the landmark information of the actuator and the number of the main cable node as the analysis object, and it is further improved through continuous analysis and verification. This ensures the accuracy and scientificity of the model to a certain extent.

## Acknowledgments

Natural Science Foundation.

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